

On the orbit of the LARES satellite

I. Ciufolini

Dipartimento di Ingegneria dell'Innovazione, Università di Lecce, Via
Monteroni, 73100 Lecce, Italy

Abstract

This paper is motivated by the recent possibility to find an inexpensive launching vehicle for the LARES satellite, however at an altitude much lower than originally planned for the LAGEOS III/LARES satellite.

We present here a preliminary error analysis corresponding to a lower, quasi-polar, orbit, in particular we analyze the effect on the LARES node of the Earth's static gravitational field, and in particular of the Earth's even zonal harmonics, the effect of the time dependent Earth's gravitational field, and in particular of the K_1 tide, and the effect of particle drag.

1 Introduction

In 1984 we proposed the use of the nodes of two laser ranged satellites of LAGEOS type to measure the Lense-Thirring effect [1, 2]. We proposed to orbit a laser-ranged satellite of LAGEOS-type (called LAGEOS III and later on LARES), with an inclination supplementary to the one of LAGEOS (launched in 1976) in order to cancel out all the secular effects on the nodes of the two laser-ranged satellites due to the deviations of the Earth's gravitational field from spherical symmetry and in particular due to the Earth's even zonal harmonics. All the other orbital parameters of LARES/LAGEOS III were proposed to be equal to the ones of LAGEOS, in particular the semi-major axis was proposed to be approximately equal to 12270 km. The mass of LAGEOS and of the proposed LAGEOS III satellite was about 400 kg.

Unfortunately, even though such an orbit of LARES would have allowed a complete cancellation of the static Earth's spherical harmonics secular effects in order to measure the much smaller Lense-Thirring effect, the weight of the proposed LARES satellite of about 400 kg and especially the high altitude of its orbit implied an expensive launching vehicle. For this reason a LARES satellite of only about 100 kg of weight was later designed [3], nevertheless the high altitude of LARES was still somehow expensive to

achieve.

Nevertheless, three new factors have changed the need of such a high altitude orbit for LARES: (a) the idea to use the nodes of N laser-ranged satellites to measure the Lense-Thirring effect and to cancel the uncertainty due to the first $N-1$ even zonal harmonics, (b) the launch of the GRACE satellite in 2002 and the publication of a new generation of very accurate Earth's gravity field models using the GRACE observations and (c) the possibility to launch the LARES satellite using an inexpensive launcher, however at a much lower altitude than originally planned.

(a) The idea to use the nodes of N satellites of LAGEOS type to cancel the effect of the first $N-1$ Earth even zonal harmonics and to measure the Lense-Thirring effect was published in 1989 [4] (see also the 1995 book [5], on page 336) as a possible alternative to the concept of the supplementary inclination satellites. This technique and in particular the idea to use the two nodes of the satellites LAGEOS and LAGEOS 2, together with the perigee of LAGEOS 2, was described in details, together with the corresponding formula, in 1996 [6]. In the 1989 paper (see also [5]), in order to measure the Lense-Thirring effect and to cancel the even zonal harmonics uncertainties, it was proposed: “For J_2 , this corresponds from formula (3.2), to an uncertainty in the nodal precession of 450 milliarcsec/year, and similarly for higher J_{2n} coefficients. Therefore the uncertainty in $\dot{\Omega}^{Lageos}$ is more than ten times larger than the Lense-Thirring precession. A solution would be to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS, to measure J_2, J_4, J_6 etc, and one satellite to measure $\dot{\Omega}^{Lense-Thirring}$ ”. At that time the error due to the even zonal harmonics was quite large due to the much less accurate Earth gravity models (available at that time) and the LAGEOS 2 satellite was not yet launched (it was launched in 1992).

This technique to use N observables to cancel the effect of the first $N-1$ even zonal harmonics was explicitly described in [6] (see also the explicit calculations about the use of the nodes of N laser-ranged satellites in [18]) and led to the publication of the detection of the Lense-Thirring effect using the LAGEOS and LAGEOS 2 satellites and the gravity field model EGM96 [7, 19] (using the nodes of LAGEOS and LAGEOS 2 and the perigee of LAGEOS 2 in order to cancel the error in the first two even zonal harmonics by using three “observables”, including the perigee, which however introduced relatively large errors due to its unmodelled non-gravitational perturbations) and to the 2004 measurement [8, 9] (with accuracy of the order of about 10 %) of the Lense-Thirring effect using the LAGEOS satellites and the accurate Earth's gravity field model EIGEN-GRACE02S, published by the GeoForschungsZentrum of Potsdam (GFZ), Germany, using the data of the

GRACE satellite. In this 2004 paper we used the same technique of the 1998 paper but we did not use the perigee of LAGEOS II (that the authors of [8] tried for a long time to avoid since the publishing of their 1997-1998 papers) thanks to the new generation of GRACE Earth's gravity models. The 2004-measurement is just the case of $N=2$ described in the abovementioned 1989 paper and it uses the nodes of the two laser ranged satellites LAGEOS and LAGEOS 2 in order to cancel the effect of the first even zonal harmonic coefficient J_2 of Earth and to measure the Lense-Thirring effect (the explicit expression of this combination was also given in [10]).

(b) The 2004 accurate measurement of the Lense-Thirring effect was possible thanks to the launch of the GRACE satellite and the publication of its accurate gravity field models by GFZ [14] and Center for Space Research (CSR) of the University of Texas at Austin. The use of the GRACE-derived gravitational models, when available, to measure the Lense-Thirring effect with accuracy of a few percent was, since almost a decade ago, a well known possibility to all the researchers in this field and was presented (and published) at several meetings by Pavlis [11] and by Ries et al. (see, e.g., [12]).

(c) In 2004, A. Paolozzi of "Scuola d'Ingegneria Aerospaziale" of the University of Rome "La Sapienza" discovered the possibility to use an inexpensive launcher to orbit LARES [13]. However, this inexpensive launch for LARES should be at a much lower altitude than the originally planned satellite at 12270 km and should be in a nearly polar orbit. The altitude achievable with this launch should be between about 1000 km and 2000 km. In 2005, J. Ries [15] informed us that CSR had done some simulations supporting this possibility of a lower orbit laser-ranged satellite. This was also suggested to us by P. Bender [16].

In the following we shall investigate on the possible orbit of the LARES satellite in relation to the error in the measurement of the Lense-Thirring effect.

2 Preliminary error analysis for a quasi-polar laser-ranged satellite at an altitude of about 1500 km

The simplest conceivable orbit in order to cancel the effect of all the even zonal harmonics on the node of a satellite would be a polar orbit, indeed for such an orbit the effect of the even zonal harmonics on the satellite node would be zero and, however, the node of the satellite would be still perturbed by the Earth's gravitomagnetic field, i.e., would be affected by the Lense-Thirring effect.

Unfortunately, as pointed out in the 1989 LAGEOS III NASA/ASI study [17]) and explicitly calculated by Peterson (1997) (chapter 5 of [18]) the uncertainty in the K_1 tide (tesseral, $m = 1$, tide) would make such an orbit unsuitable for the Lense-Thirring measurement. Indeed, a polar satellite would have a secular precession of its node whose uncertainty would introduce a large error in the Lense-Thirring measurement. In addition, it would be quite demanding to launch LARES with the requirement of a small orbital injection error from a polar orbit (even at a lower altitude than the one planned for LAGEOS III, for a measurement error of about 1 % due to the uncertainties in the static Earth gravity field, the deviation from a polar orbit should be less than about 0.1 degrees).

Nevertheless, a quasi-polar orbit would have a nodal precession, due to its departure from 90 degrees of inclination, and thus one could simply fit for the effect of the K_1 tide using a periodical signal exactly at the nodal frequency. Such frequency (with the period of the LAGEOS satellites node) is indeed observed in the LAGEOS 1 and LAGEOS 2 analyses already mentioned [7, 8] and is the largest periodical amplitude observed in the combined residuals.

If we assume that the LARES orbit would have an altitude of 1500 km, then, by imposing for example that the minimum period of observation in order to measure the Lense-Thirring effect should not be longer than three years, we have that the LARES inclination should be less than or equal to 86 degrees or larger than or equal to 94 degrees.

In regard to the effect of the static even zonal harmonics, by using the technique explained in [4, 6] and by using the nodes of the satellites LARES, LAGEOS and LAGEOS 2, we would be able to cancel the uncertainties due to the first two even zonal harmonics, C_{20} and C_{40} , and our measurement will only be affected by the uncertainties due to the even zonal harmonics with degree strictly higher than 4.

By solving the system of the three equations for the nodal precessions of LAGEOS, LAGEOS I and LARES in the three unknowns, J_2 , J_4 and Lense-Thirring effect, we have a combination of three observables (the three nodal rates) which determines the Lense-Thirring effect independently of any uncertainty δC_{20} and δC_{40} in the first two even zonal harmonics. This same technique was applied in [7] using the nodes of LAGEOS and LAGEOS 2 and the perigee of LAGEOS 2 and in [8] using the nodes of LAGEOS and LAGEOS 2 only.

It turns out that some values of the inclination of LARES would minimize the error in the measurement of the Lense-Thirring effect since they would minimize the error due to the uncertainty in the largest (not cancelled using the combination of the three observables) even zonal harmonic C_{60} .

In figure 1 we have plotted the error in the measurement of the Lense-Thirring effect, using LARES, LAGEOS and LAGEOS 2, as a function of the inclination and of the semimajor axis. The range of the altitude of LARES is between 1000 km and 2000 km and of the inclination between 0 and 360 degrees, of course if LARES would be launched in a nearly polar orbit the use of the LAGEOS and LAGEOS 2 satellites would not be anymore useful in order to reduce the error budget (and would indeed only introduce an additional error), since the effect of the even zonal harmonics on the node of LARES would be nearly zero, however, as previously remarked, the measurement of the Lense-Thirring effect using a polar orbit would be substantially affected by the uncertainty in the K_1 tide.

In figure 2 we have plotted the error in the measurement of the Lense-Thirring effect as a function of the inclination by assuming an altitude of LARES of 1500 km, i.e., a LARES semimajor axis of about 7870 km.

From Figure 2 we can see that any inclination from 60 degrees to 86 degrees and from 94 to 120 degrees would be suitable for a measurement of the Lense-Thirring effect with accuracy of a few percent. An inclination of LARES of about 110 degrees or 70 degrees would minimize the error. In deriving this result, we have assumed: (a) zero eccentricity for the LARES orbit, (b) we have only considered the effect of the first 5 even zonal harmonics [20]: C_{20} , C_{40} , C_{60} , C_{80} and C_{100} and (c) we have considered the uncertainties in the spherical harmonics C_{60} , C_{80} and C_{100} to be equal to those of the EIGEN-GRACE02S Earth's gravity model [14], i.e., we have assumed $\delta C_{60} = 0.2049 \cdot 10^{-11}$ $\delta C_{80} = 0.1479 \cdot 10^{-11}$ $\delta C_{100} = 0.2101 \cdot 10^{-11}$. Nevertheless, by including higher degree even zonal harmonics, by considering an eccentricity different from zero, e.g., equal to 0.01, and by considering an error a priori equal for these three spherical harmonics, i.e. $\delta C_{60} = \delta C_{80} = \delta C_{100} = 0.2 \cdot 10^{-11}$, Figures 1 and 2 would not appreciably change and our results would still remain valid. This uncertainty, $0.2 \cdot 10^{-11}$, is nearly the uncertainty in the even zonal coefficients of the EIGEN-GRACE02S model (used in [8]) and given above); indeed, even though the real error in these coefficients would probably be about two times larger than these published values, these uncertainties refer to a preliminary 2004 model and by the time of the launch of LARES and of its data analysis (about 2008-2011), Earth's gravity field models much more accurate based on much longer data set of GRACE observations would be available.

In regard to the other orbital perturbations that affect the LARES experiment we briefly discuss here the tidal effects, particle drag and thermal drag; for a detailed total error budget we refer to [4, 17, 22]. In regard to the

orbital perturbations on the LARES experiment due to the time dependent Earth's gravity field, we observe that the largest tidal signals are due to the zonal tides with $l = 2$ and $m = 0$, due to the Moon node, and to the K_1 tide with $l = 2$ and $m = 1$ (tesseral tide). However, the medium and long period zonal tides ($l = 2$ and $m = 0$) will be cancelled using the combination of the three nodes together with the static C_{20} uncertainty (also the uncertainty in the time-dependent secular variations \dot{C}_{20} , \dot{C}_{40} will be cancelled using this combination of three observables). Furthermore, the tesseral tide K_1 will be fitted for over a period equal to the LARES nodal period as explained above (see [17] and chapter 5 of [18]) and this tide would then introduce a small uncertainty in our combination. In regard to the non-gravitational orbital perturbations, we observe here that the unmodelled thermal drag perturbations on the LARES orbit would be reduced thanks to the accurate measurements of the thermal properties of the LARES satellite and of its retro-flectors that are performed by the group of S. Dell'Agnello at the Laboratori Nazionali di Frascati of INFN [23]. We finally point out that the neutral and charged particle drag on the LARES node at an altitude of about 1500 km would be a negligible effect for an orbit with very small eccentricity, even by assuming that the exosphere would be co-rotating with Earth at 1500 km of altitude. Indeed, as calculated in [4] for the LAGEOS III satellite, in the case of zero orbital eccentricity $e = 0$ the total drag effect on the LARES node would be zero; indeed the nodal rate of a satellite due to particle drag is a function of $\sin \nu \cdot \cos \nu$ (ν is the true anomaly) and the total nodal shift is then zero over one orbit; in the case of a small orbital eccentricity, the total shift would be proportional to the eccentricity and it would still be a small effect as calculated in [4].

3 Conclusion

A nearly polar orbit for LARES at an altitude of about 1500 km would be suitable for a measurement of the Lense-Thirring effect with accuracy of a few percent. Some values of the inclination of LARES would minimize the measurement error induced by the uncertainties in the even zonal harmonics. An inclination off the polar one by about 4 degrees would allow the average and the fit of the K_1 tidal uncertainty over a period of about 3 years.

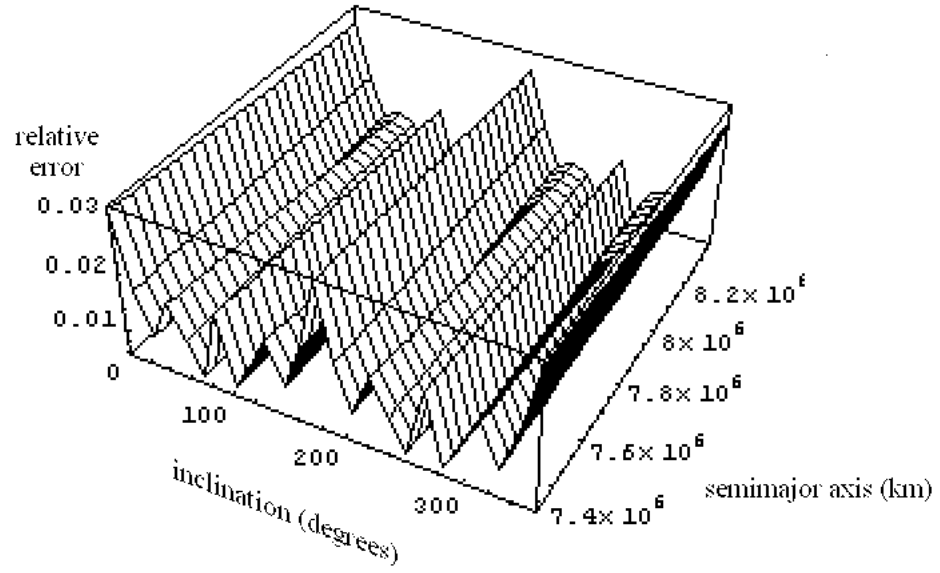


Figure 1: Uncertainty in the measurement of the Lense-Thirring effect, due to the even zonal harmonics uncertainties, as a function of the inclination and of the semimajor axis of LARES, using LARES, LAGEOS and LAGEOS 2. The range of the altitude of LARES is between 1000 km and 2000 km and of the inclination between 0 and 360 degrees

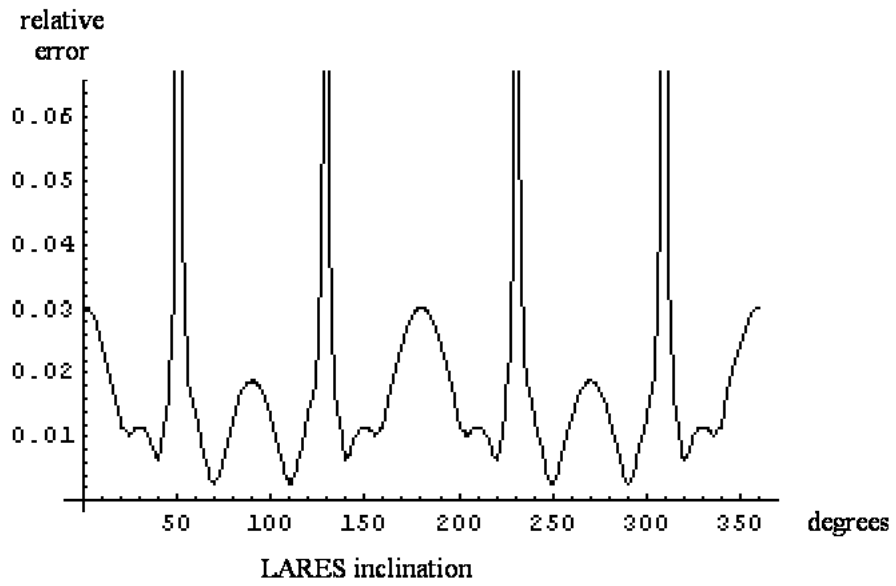


Figure 2: Uncertainty in the measurement of the Lense-Thirring effect, due to the even zonal harmonics uncertainties, as a function of the inclination of LARES, using LARES, LAGEOS and LAGEOS 2. The altitude of LARES is here 1500 km and the range of the inclination between 0 and 360 degrees

References

- [1] I. Ciufolini, *Theory and Experiments in General Relativity and other Metric Theories*, Ph. Dissertation, Univ. of Texas, Austin, Ann Arbor, Michigan (1984).
- [2] I. Ciufolini, *Phys. Rev. Lett.* **56**, 278 (1986).
- [3] I. Ciufolini, A. Paolozzi, E. Pavlis et al. *LARES phase A study for ASI* (1998).
- [4] I. Ciufolini, *Int. J. Mod. Phys. A* **4**, 3083 (1989).
- [5] I. Ciufolini and J.A. Wheeler, *Gravitation and Inertia* (Princeton University Press, Princeton, New Jersey, 1995).
- [6] I. Ciufolini, *Nuovo Cimento A* **109**, 1709 (1996).
- [7] I. Ciufolini, E.C. Pavlis, F. Chieppa, E. Fernandes-Vieira and J. Perez-Mercader, *Science* **279**, 2100 (1998).
- [8] I. Ciufolini and E. Pavlis, *Letters to Nature*, **431**, 958 (2004).
- [9] I. Ciufolini, E. Pavlis and R. Peron, *New Astronomy* **11**, 527 (2006).
- [10] I. Ciufolini, proceedings of the SIGRAV General Relativity and Gravitation School, Villa Mondragone, Frascati, 2002, IOP (2004).
- [11] E. C. Pavlis, *Geodetic Contributions to Gravitational Experiments in Space*, in Recent Developments in General Relativity, Genoa 2000, R. Cianci, R. Collina, M. Francaviglia, P. Fr (eds), pp. 217-233, Springer-Verlag, Milan, (2002).
- [12] J. C. Ries, R. J. Eanes, B. D. Tapley and G. E. Peterson, in Toward Millimeter Accuracy Proc. 13th Int. Laser Ranging Workshop, Noomen, R., Klosko, S., Noll, C. and Pearlman, M. eds., (NASA CP 2003212248, NASA Goddard, Greenbelt, MD, 2003).
- [13] A. Paolozzi, Private communication (2004).
- [14] Reigber, C., Schmidt, R., Flechtner, F., Knig, R., Meyer, U., Neumayer, K.H., Schwintzer, P., Zhu, S.Y. An Earth gravity field model complete to degree and order 150 from GRACE: EIGEN-GRACE02S, *J. Geodyn.* **39**, 1 (2005). The EIGEN-GRACE02S gravity field coefficients and their calibrated errors are available at: http://op.gfz-potsdam.de/grace/index_GRACE.html 18.

- [15] J. Ries, Private communication (2005).
- [16] P. Bender, Private communication (2006).
- [17] B. Tapley, I. Ciufolini, J.C. Ries, R.J. Eanes, M.M. Watkins, *NASA-ASI Study on LAGEOS III*, CSR-UT publication n. CSR-89-3, Austin, Texas (1989). See also: I. Ciufolini et al., INFN study on LARES/WEBER-SAT (2004).
- [18] G. E. Peterson, *Estimation of the Lense-Thirring precession using laser-ranged satellites*, Ph. Dissertation, Univ. of Texas, Austin, 1997.
- [19] I. Ciufolini, F. Chieppa, D. Lucchesi and F. Vespe, *Class. and Quantum Grav.* **14**, 2701 (1997).
- [20] The analytical expression of the coefficients of the nodal precession of a satellite has been provided us, up to $l = 10$, by R. Tauraso (2004) on the basis of the formulas given by Kaula 1966 [21] and for $l = 2$ and $l = 4$ was published in [6].
- [21] W.M. Kaula, *Theory of Satellite Geodesy* (Blaisdell, Waltham, 1966).
- [22] I. Ciufolini, A. Paolozzi, S. Dell’Agnello, E.C. Pavlis, G. Sindoni and R.Tauraso, in preparation (2006).
- [23] A. Bosco et al., *Int. J. Mod. Phys. D* in print (2006).